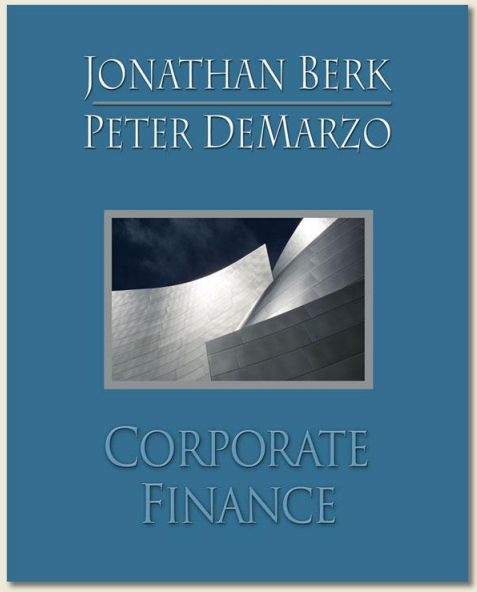


Chapter 5

Valuing Bonds

Chapter Outline

5.1 Bond Cash Flows, Prices, and Yields

- Bond Terminology
  - Bond Certificate
    - States the terms of the bond
  - Maturity Date
    - Final repayment date
  - Term
    - The time remaining until the repayment date
  - Coupon
    - Promised interest payments

5.1 Bond Cash Flows, Prices, and Yields (cont'd)

- Bond Terminology
  - Face Value
    - Notional amount used to compute the interest payments
  - Coupon Rate
    - Determines the amount of each coupon payment, expressed as an APR
  - Coupon Payment

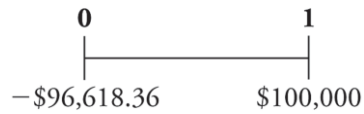
$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

Zero-Coupon Bonds

- Zero-Coupon Bond
  - Does not make coupon payments
  - Always sells at a **discount** (a price lower than face value), so they are also called **pure discount bonds**
  - **Example: Treasury Bills** are U.S. government zero-coupon bonds with a maturity of up to one year.

Zero-Coupon Bonds (cont'd)

- Suppose that a one-year, risk-free, zero-coupon bond with a $100,000 face value has an initial price of $96,618.36. The cash flows would be:



  - Although the bond pays no “interest,” your compensation is the difference between the initial price and the face value.

Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - *The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.*
    - *Price of a Zero-Coupon bond*

$$P = \frac{FV}{(1 + YTM_n)^n}$$

Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - For the one-year zero coupon bond:

$$96{,}618.36 = \frac{100{,}000}{(1 + YTM_1)}$$

$$1 + YTM_1 = \frac{100{,}000}{96{,}618.36} = 1.035$$

- Thus, the YTM is 3.5%.

Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - Yield to Maturity of an *n*-Year Zero-Coupon Bond

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1$$

Example 8.1

Yields for Different Maturities

Problem

Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

Maturity	1 year	2 years	3 years	4 years
Price	$96.62	$92.45	$87.63	$83.06

Example 8.1 (cont'd)

Solution

Using Eq. 8.3, we have

$$YTM_1 = (100 / 96.62) - 1 = 3.50\%$$
$$YTM_2 = (100 / 92.45)^{1/2} - 1 = 4.00\%$$
$$YTM_3 = (100 / 87.63)^{1/3} - 1 = 4.50\%$$
$$YTM_4 = (100 / 83.06)^{1/4} - 1 = 4.75\%$$

Zero-Coupon Bonds (cont'd)

- Risk-Free Interest Rates
  - A default-free zero-coupon bond that matures on date *n* provides a risk-free return over the same period. Thus, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.
  - Risk-Free Interest Rate with Maturity *n*

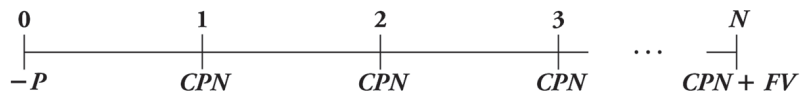
$$r_n = YTM_n$$

Zero-Coupon Bonds (cont'd)

- Risk-Free Interest Rates
  - Spot Interest Rate
    - Another term for a default-free, zero-coupon yield
  - Zero-Coupon Yield Curve
    - A plot of the yield of risk-free zero-coupon bonds as a function of the bond's maturity date

Coupon Bonds (cont'd)

- Yield to Maturity
  - The YTM is the *single* discount rate that equates the present value of the bond's remaining cash flows to its current price.



  - Yield to Maturity of a Coupon Bond

$$P = CPN \times \frac{1-(1+y)^{-N}}{y} + \frac{FV}{(1+y)^{N}}$$

5.2 Dynamic Behavior of Bond Prices

- Discount
  - A bond is selling at a **discount** if the price is less than the face value.
- Par
  - A bond is selling at **par** if the price is equal to the face value.
- Premium
  - A bond is selling at a **premium** if the price is greater than the face value.

Discounts and Premiums

- If a coupon bond trades at a discount, an investor will earn a return both from receiving the coupons and from receiving a face value that exceeds the price paid for the bond.
  - If a bond trades at a discount, its yield to maturity will exceed its coupon rate.

Discounts and Premiums (cont'd)

- If a coupon bond trades at a premium it will earn a return from receiving the coupons but this return will be diminished by receiving a face value less than the price paid for the bond.
- Most coupon bonds have a coupon rate so that the bonds will *initially* trade at, or very close to, par.

Discounts and Premiums (cont'd)

TABLE 8.1 Bond Prices Immediately After a Coupon Payment

When the bond price is . . .	greater than the face value	equal to the face value	less than the face value
We say the bond trades	"above par" or "at a premium"	"at par"	"below par" or "at a discount"
This occurs when	Coupon Rate > Yield to Maturity	Coupon Rate = Yield to Maturity	Coupon Rate < Yield to Maturity

Interest Rate Changes and Bond Prices

- There is an inverse relationship between interest rates and bond prices.
  - As interest rates and bond yields rise, bond prices fall.
  - As interest rates and bond yields fall, bond prices rise.

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5.3 The Yield Curve and Bond Arbitrage

- Using the Law of One Price and the yields of default-free zero-coupon bonds, one can determine the price and yield of any other default-free bond.
- The yield curve provides sufficient information to evaluate all such bonds.

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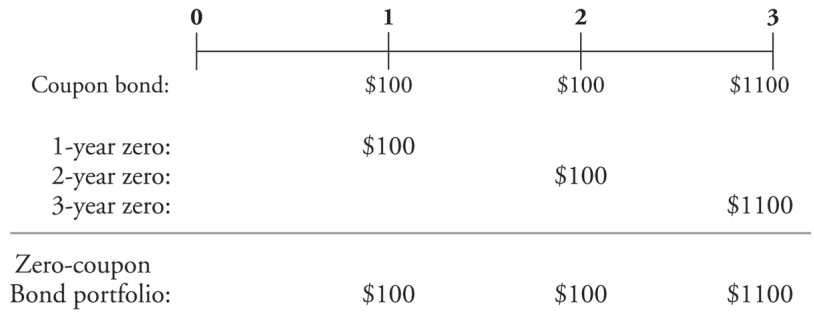
Replicating a Coupon Bond

- Replicating a three-year $1000 bond that pays 10% annual coupon using three zero-coupon bonds:



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Replicating a Coupon Bond (cont'd)

- Yields and Prices (per $100 Face Value) for Zero Coupon Bonds

TABLE 8.2 **Yields and Prices (per $100 Face Value) for Zero-Coupon Bonds**

Maturity	1 year	2 years	3 years	4 years
YTM	3.50%	4.00%	4.50%	4.75%
Price	$96.62	$92.45	$87.63	$83.06

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Replicating a Coupon Bond (cont'd)

Zero-Coupon Bond	Face Value Required	Cost
1 year	100	96.62
2 years	100	92.45
3 years	1100	$11 \times 87.63 = 963.93$
	Total Cost:	$1153.00

- By the Law of One Price, the three-year coupon bond must trade for a price of $1153.

Valuing a Coupon Bond Using Zero-Coupon Yields

- The price of a coupon bond must equal the present value of its coupon payments and face value.
  - Price of a Coupon Bond

$$PV = PV(\text{Bond Cash Flows}) = \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \cdots + \frac{CPN + FV}{(1 + YTM_n)^n}$$

$$P = \frac{100}{1.035} + \frac{100}{1.04^2} + \frac{100 + 1000}{1.045^3} = \$1153$$

Treasury Yield Curves

- Treasury Coupon-Paying Yield Curve
  - Often referred to as “the yield curve”
- Coupon Bonds
  - Most recently issued bonds
  - The yield curve is often a plot of the yields on these bonds.

5.4 Corporate Bonds

- Corporate Bonds
  - Issued by corporations
- Credit Risk
  - Risk of default

Corporate Bond Yields

- Investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond.
- The yield of bonds with credit risk will be higher than that of otherwise identical default-free bonds.

Corporate Bond Yields (cont'd)

- No Default
  - Consider a 1-year, zero coupon Treasury Bill with a YTM of 4%.
    - What is the price?

$$P = \frac{1000}{1 + YTM_1} = \frac{1000}{1.04} = \$961.54$$

Corporate Bond Yields (cont'd)

- Certain Default
  - Suppose now bond issuer will pay 90% of the obligation.
    - What is the price?

$$P = \frac{900}{1 + YTM_1} = \frac{900}{1.04} = \$865.38$$

Corporate Bond Yields (cont'd)

- Certain Default
  - When computing the yield to maturity for a bond with certain default, the promised rather than the actual cash flows are used.

$$YTM = \frac{FV}{P} - 1 = \frac{1000}{865.38} - 1 = 15.56\%$$

$$\frac{900}{865.38} = 1.04$$

Corporate Bond Yields (cont'd)

- Certain Default
  - The yield to maturity of a certain default bond is not equal to the expected return of investing in the bond. The yield to maturity will always be higher than the expected return of investing in the bond.

Corporate Bond Yields (cont'd)

- Risk of Default
  - Consider a one-year, $1000, zero-coupon bond issued. Assume that the bond payoffs are uncertain.
    - There is a 50% chance that the bond will repay its face value in full and a 50% chance that the bond will default and you will receive $900. Thus, you would expect to receive $950.
    - Because of the uncertainty, the discount rate is 5.1%.

Corporate Bond Yields (cont'd)

- Risk of Default
  - The price of the bond will be

$$P = \frac{950}{1.051} = \$903.90$$

  - The yield to maturity will be

$$YTM = \frac{FV}{P} - 1 = \frac{1000}{903.90} - 1 = .1063$$

Corporate Bond Yields (cont'd)

- Risk of Default
  - A bond's expected return will be less than the yield to maturity if there is a risk of default.
  - A higher yield to maturity does not necessarily imply that a bond's expected return is higher.

Bond Ratings

- Investment Grade Bonds
- Speculative Bonds
  - Also known as Junk Bonds or High-Yield Bonds

TABLE 8.4 Bond Ratings

Moody's	Standard & Poor's	Description (Moody's)
Investment Grade Debt		
Aaa	AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa	AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A	A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa	BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.

(*continues*)

Bond Ratings (cont'd)

TABLE 8.4 Bond Ratings (continued)

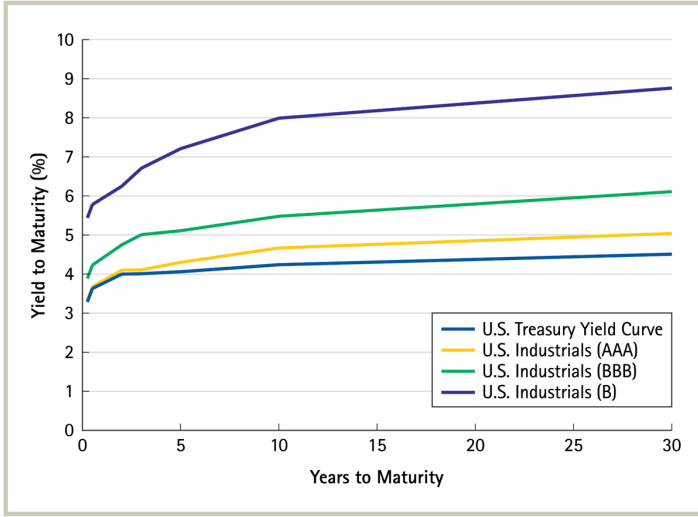
Moody's	Standard & Poor's	Description (Moody's)
Speculative Bonds		
Ba	BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B	B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa	CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca	CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C	C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

Source: www.moodys.com.

Corporate Yield Curves

- Default Spread
  - Also known as Credit Spread
  - The difference between the yield on corporate bonds and Treasury yields

Figure 8.3 Corporate Yield Curves for Various Ratings, September 2005



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